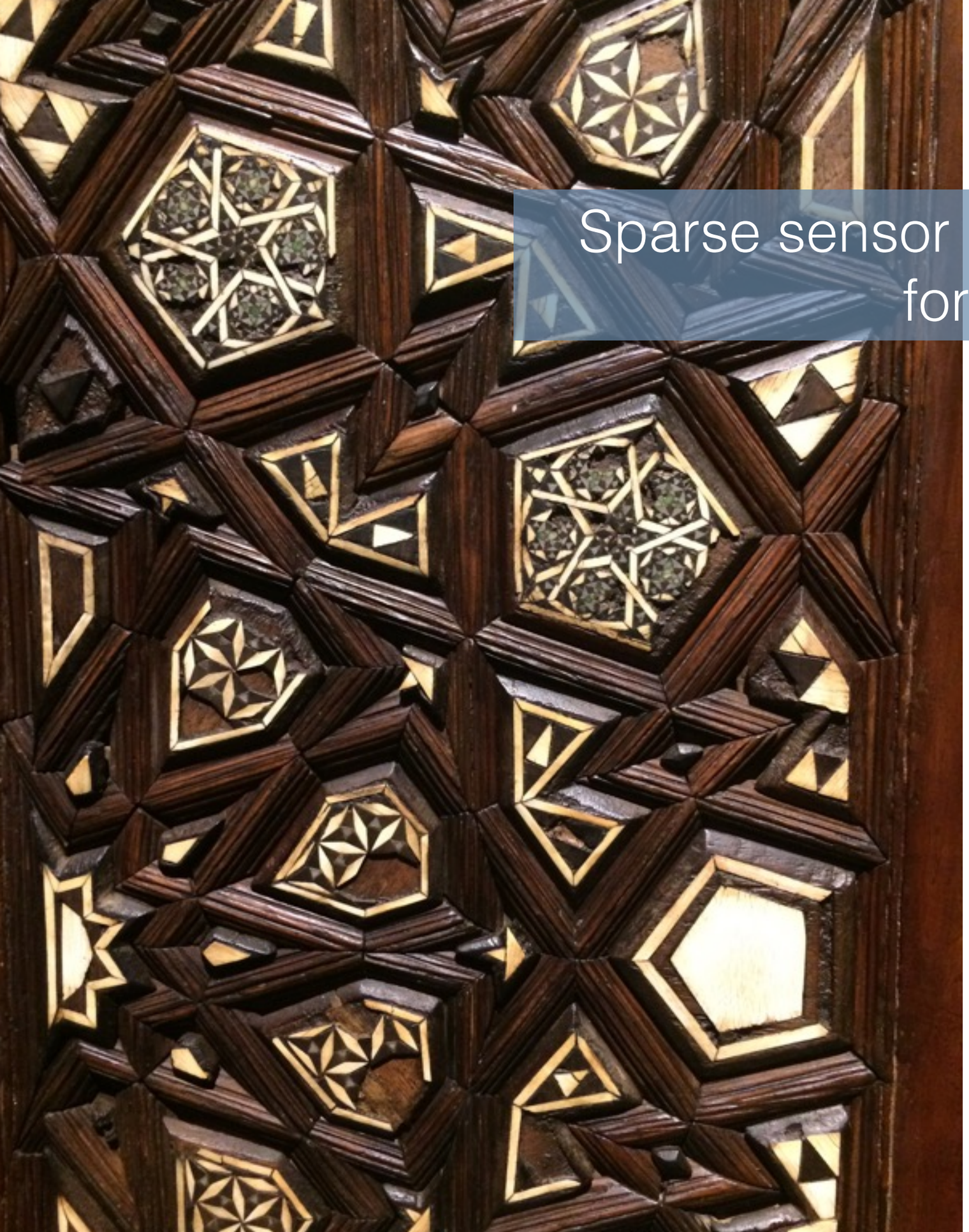


Sparse sensor placement optimization for classification (SSPOC)

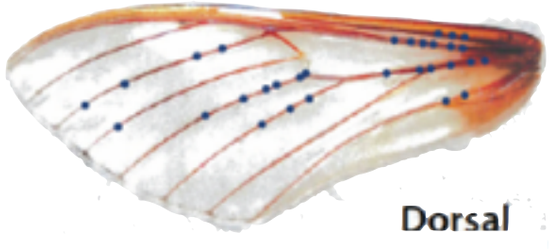
Bing W. Brunton
Washington Research Foundation
Innovation Assistant Professor

Dept. of Biology
UW Institute of Neuroengineering
Data Science Fellow, eScience Institute
Program in Neuroscience
University of Washington

2016-04-19
IDM, Disease Modeling Symposium

Given a fixed budget of sensors,
where should they be placed to optimally inform decision-making?

sensor networks in biology



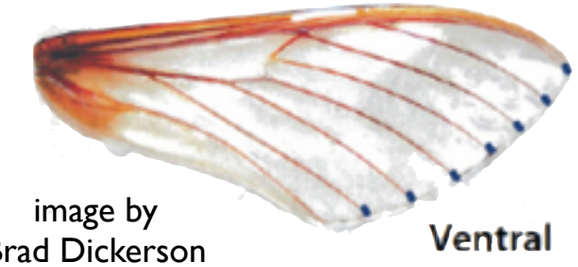
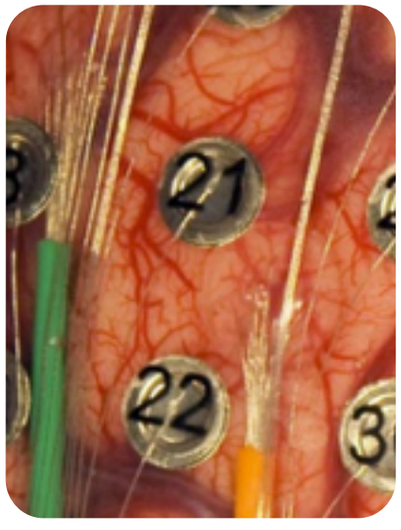


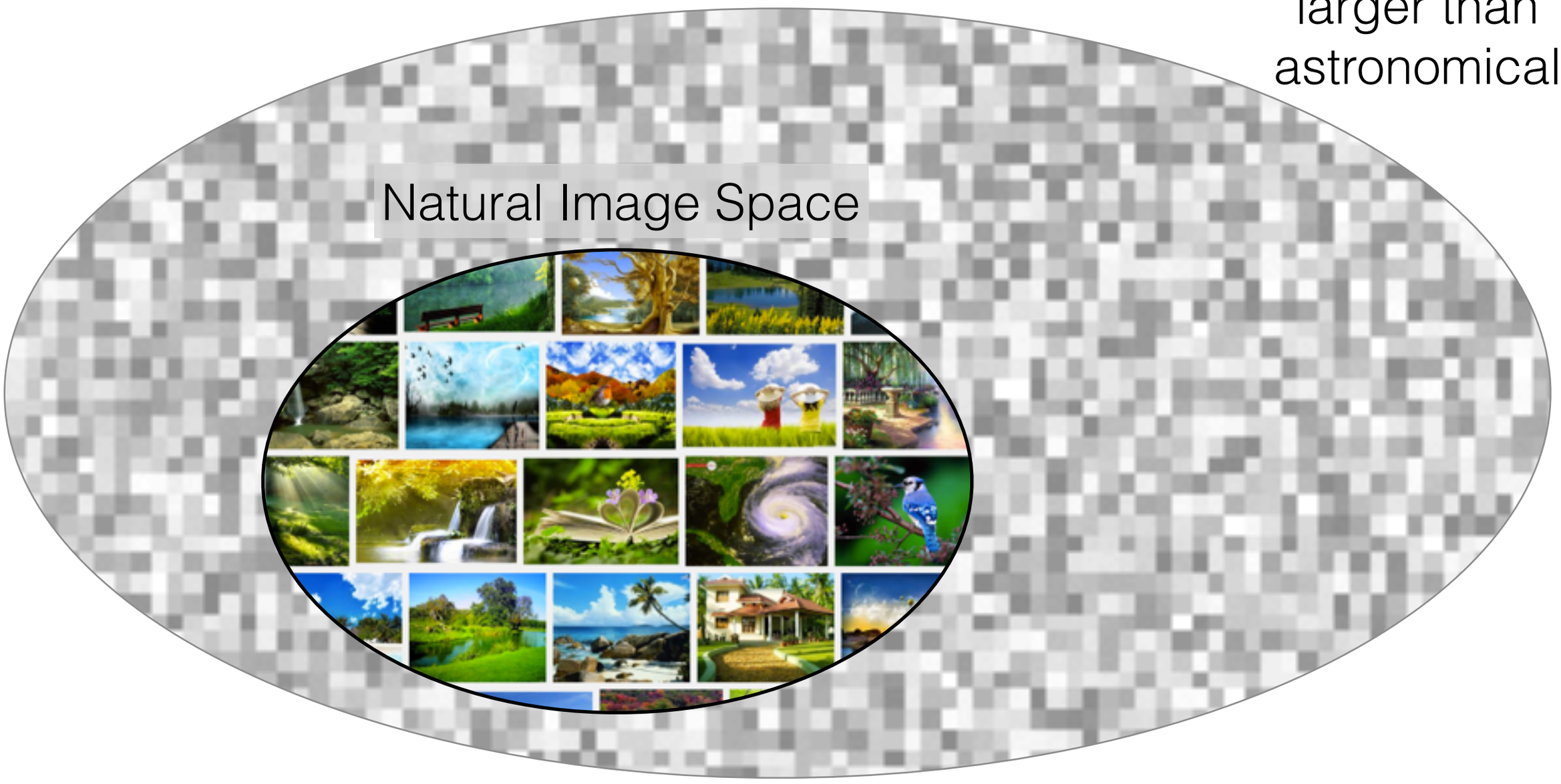
image by
Brad Dickerson

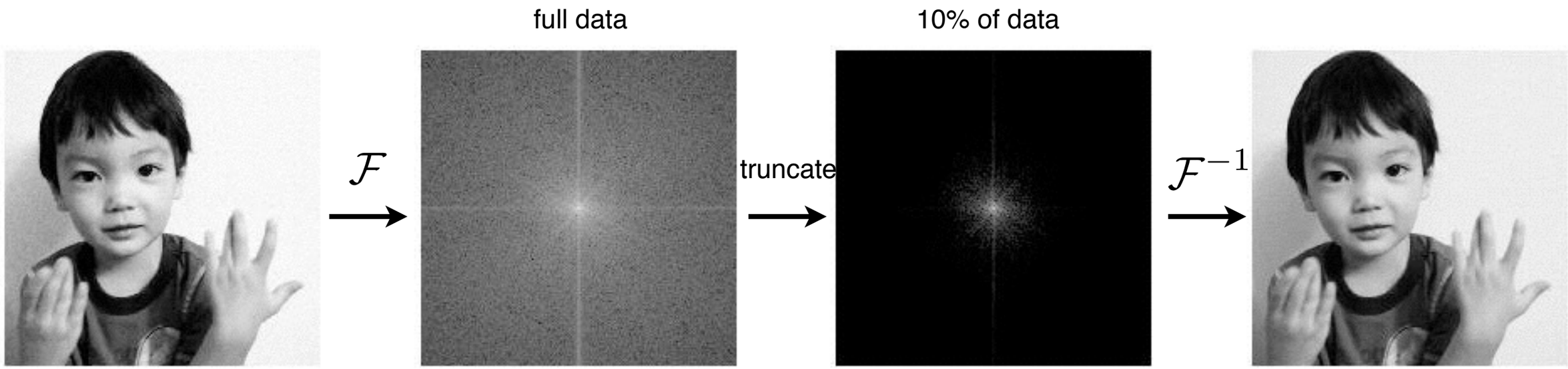


sensor networks for measurement and surveillance

Relatively simple patterns often underly complex data

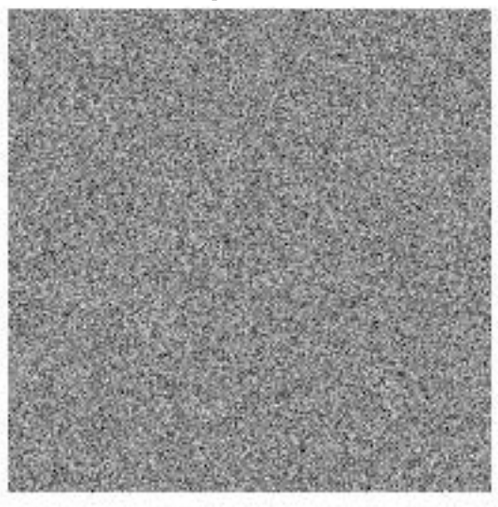
Pixel space is larger than astronomical





Compression and Compressive Sensing

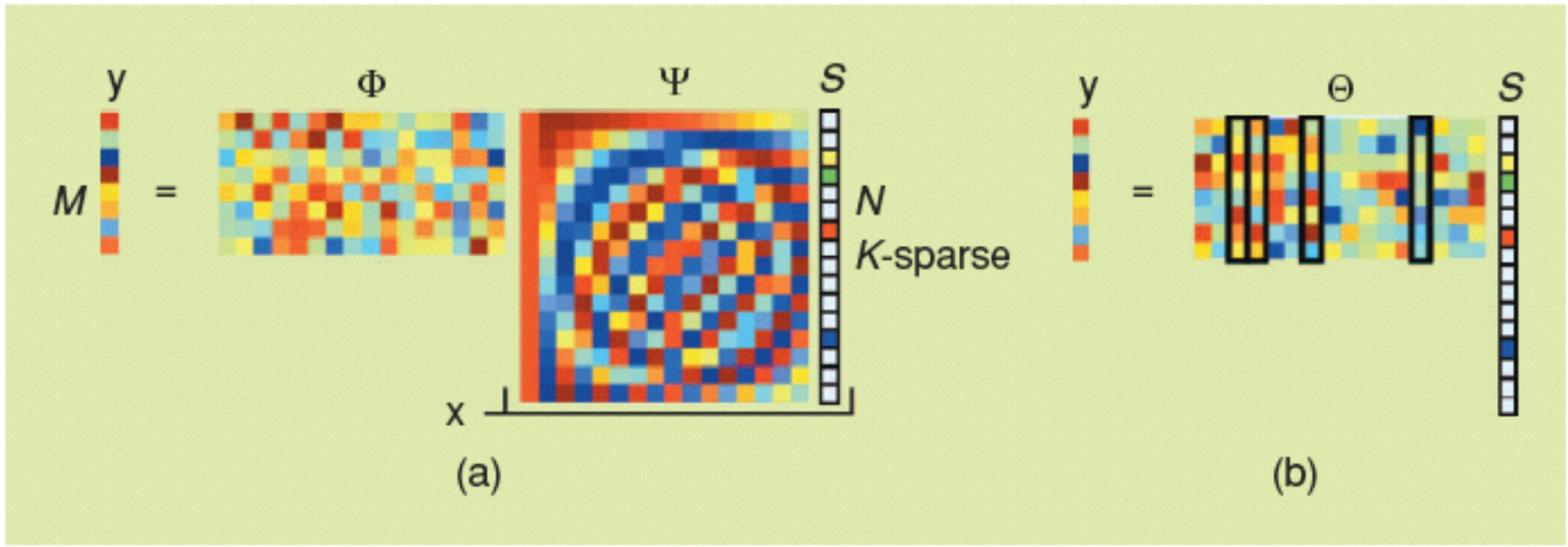
10% random† measurements



reconstruct by
solving for sparse representation

† subject to some specific constraints

Reconstruction by Compressive Sensing



from Baraniuk, 2007.

original

reconstructed from
10% measurements

single pixel camera, reconstructions from
http://dsp.rice.edu/cscamera

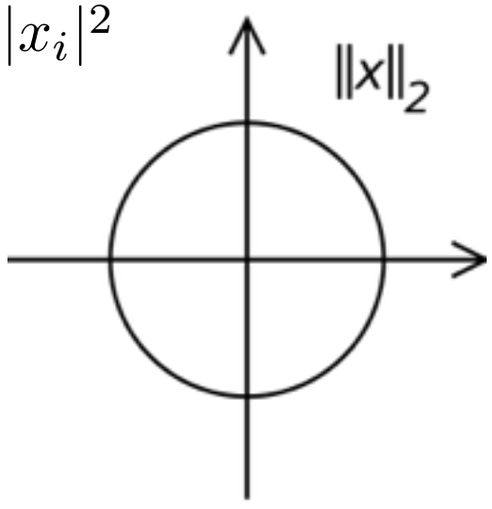
To reconstruct:

$$\text{minimize} \quad \|s\|_1,$$
$$\text{such that} \quad y = \Theta s.$$

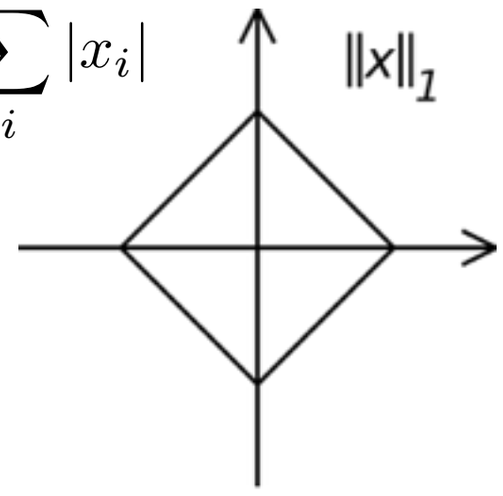
- Candès, Romberg & Tao, 2006.
- Donoho, 2006.

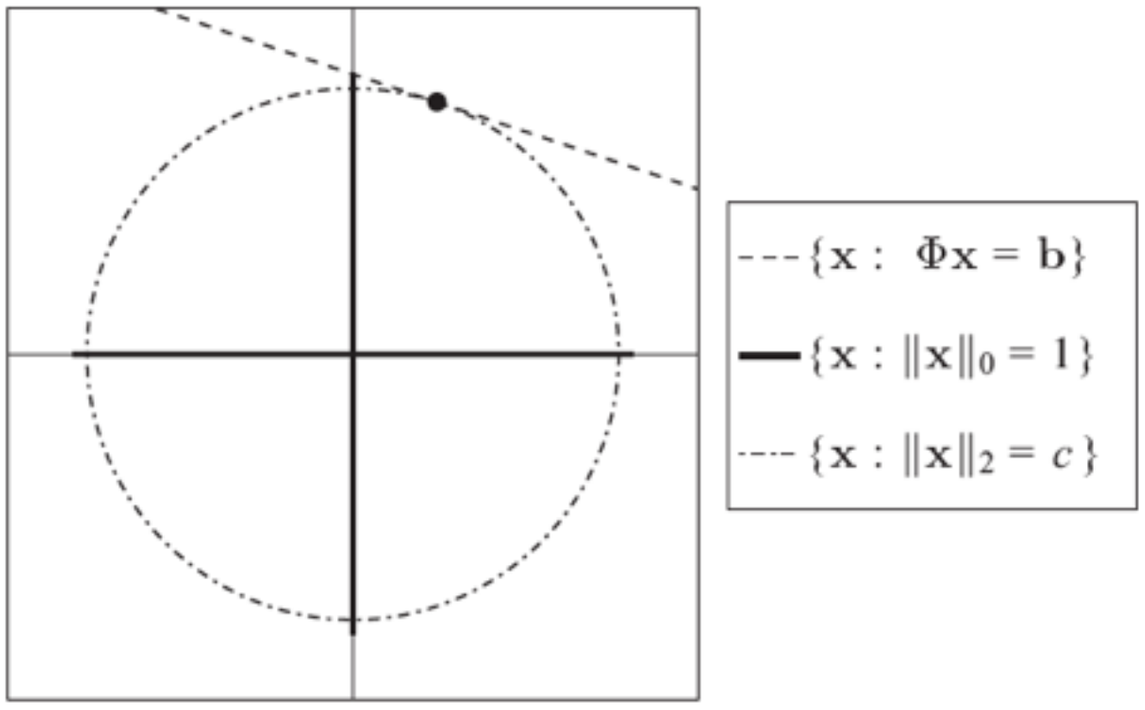
Why does l_1-minimization promote sparsity?

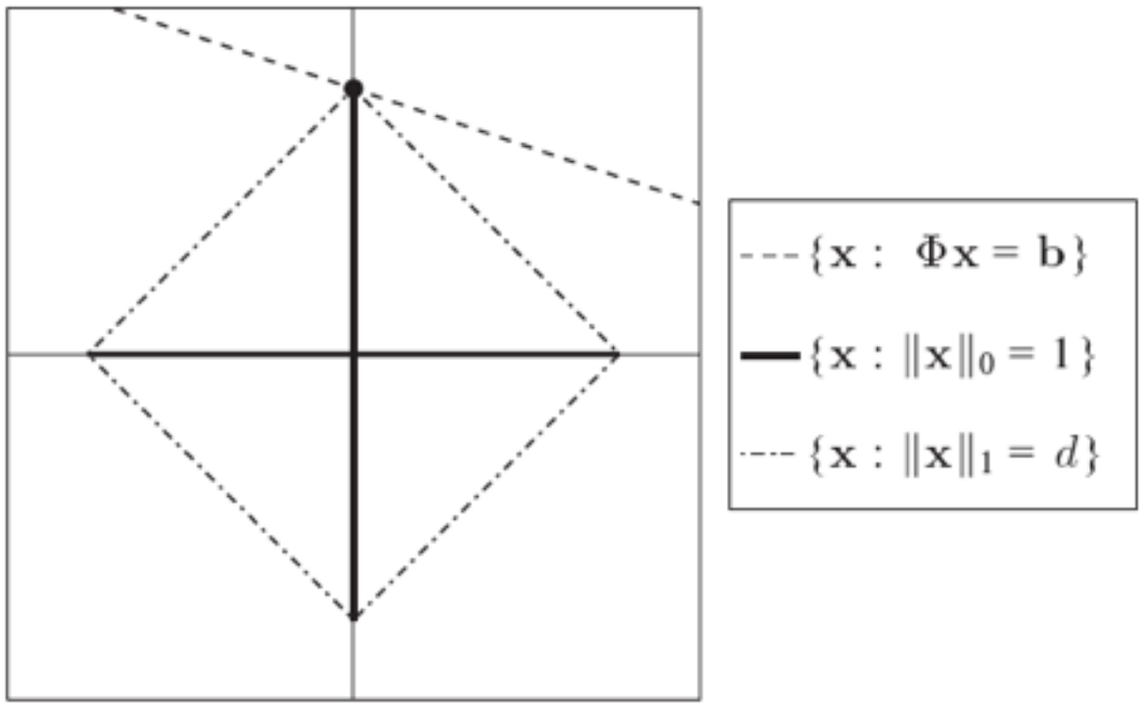
$$|\mathbf{x}|_2 = \sqrt{\sum_i |x_i|^2}$$



$$|\mathbf{x}|_1 = \sum_i |x_i|$$







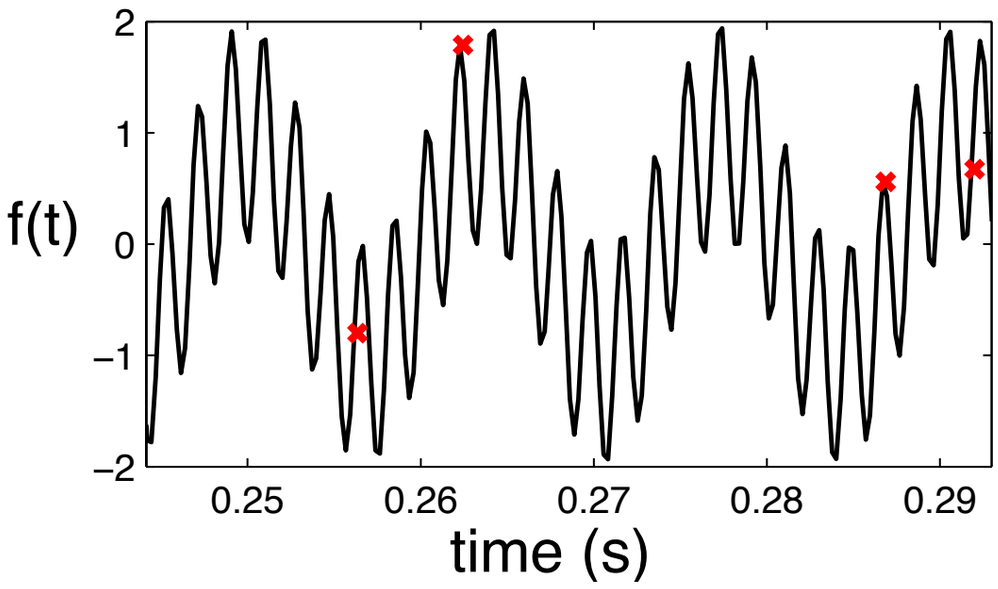
from Bryan & Leise, SIAM Review 2013

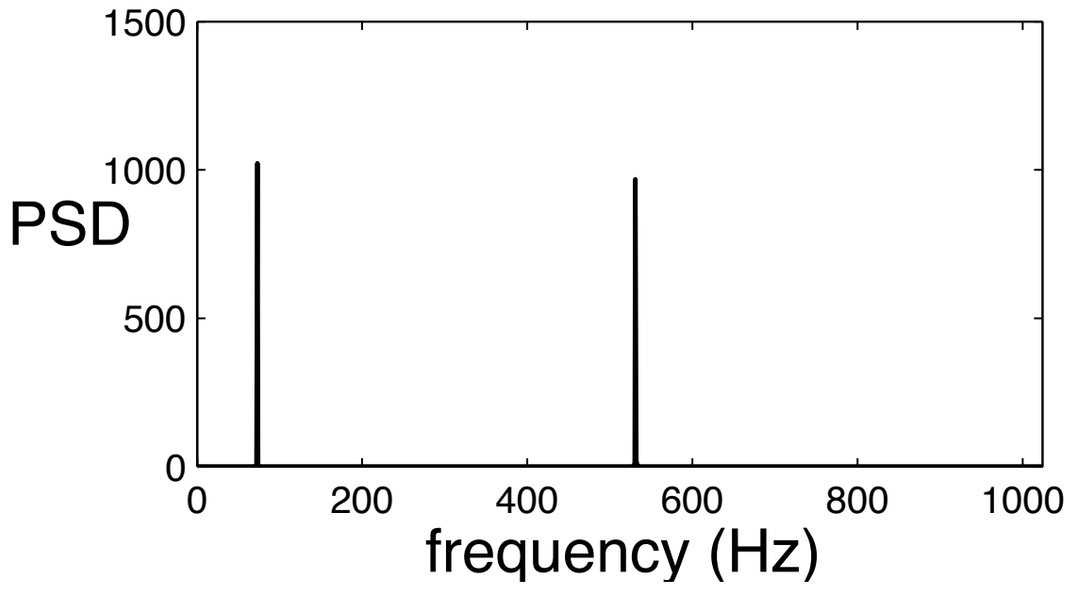
Simple Example: Beating Nyquist Sampling

$$f(t) = \sin(73 \times 2\pi t) + \sin(531 \times 2\pi t)$$

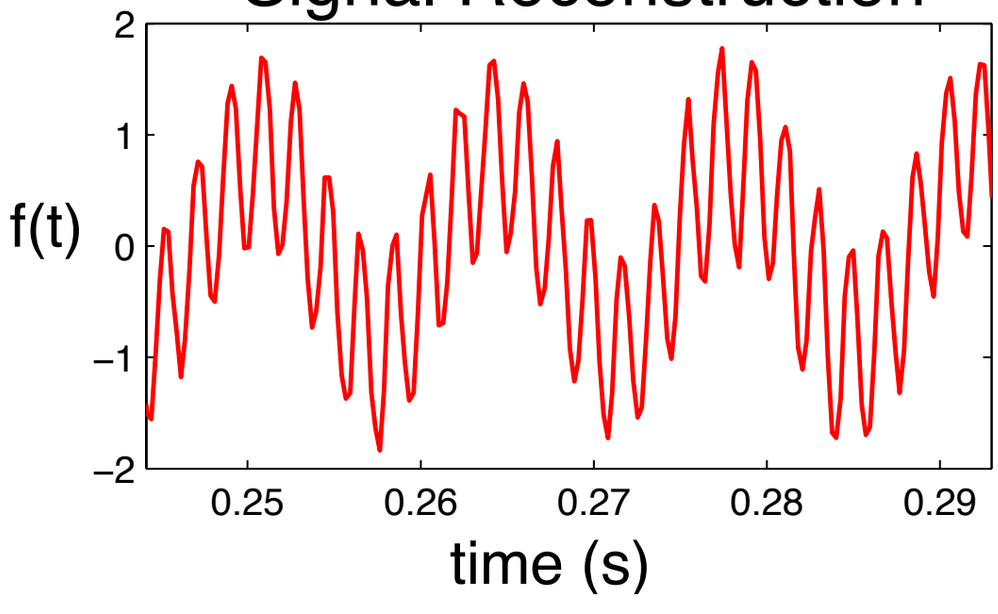
Nyquist: 1062 samples/second

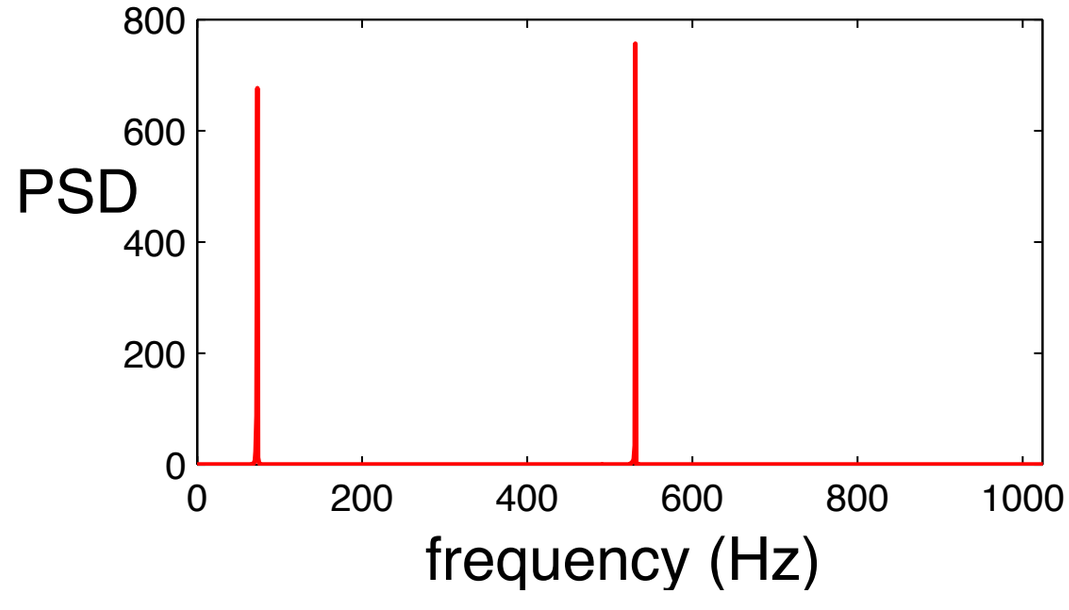
Compressed Sampling: 128 samples/second

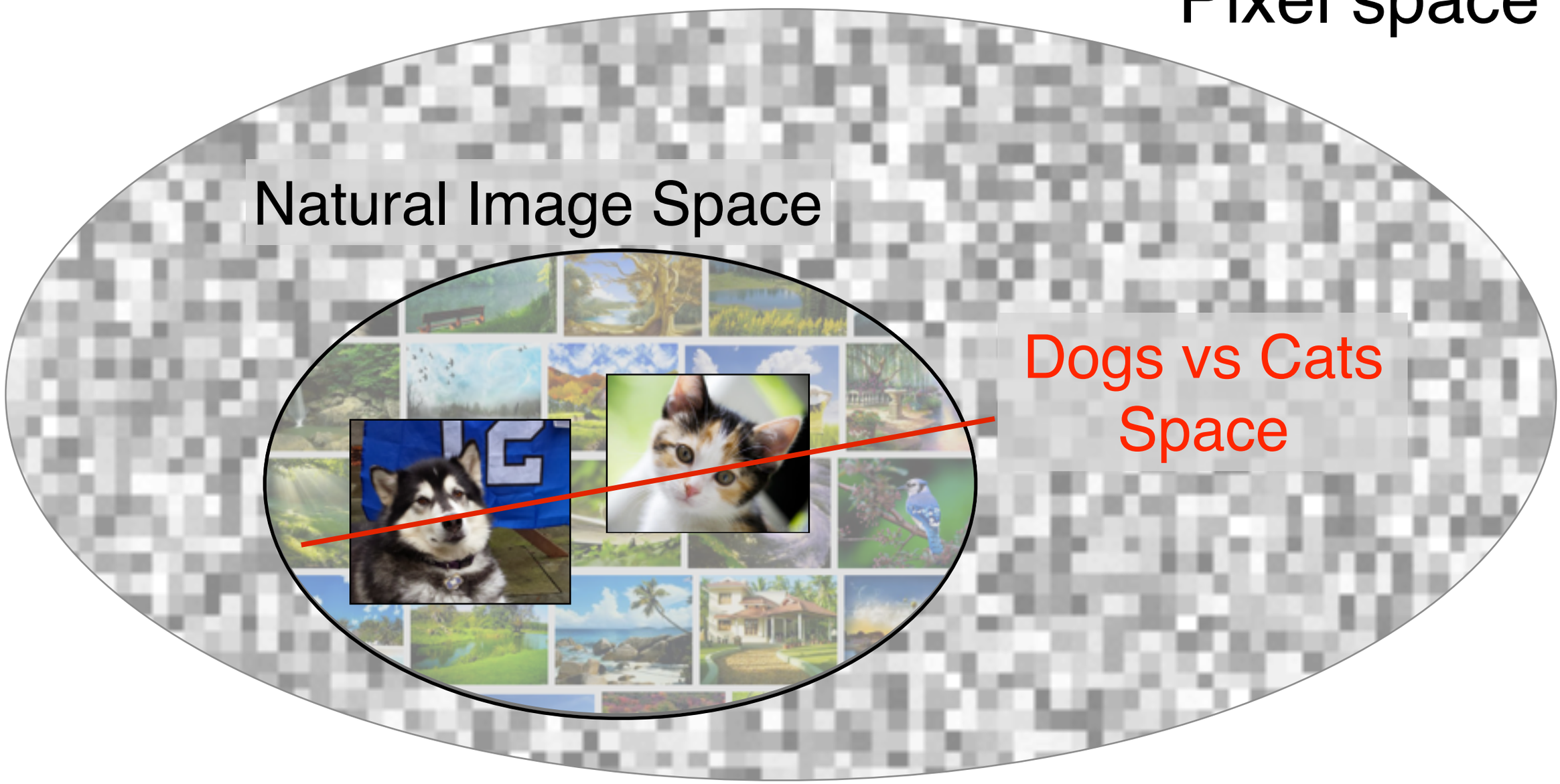




Signal Reconstruction

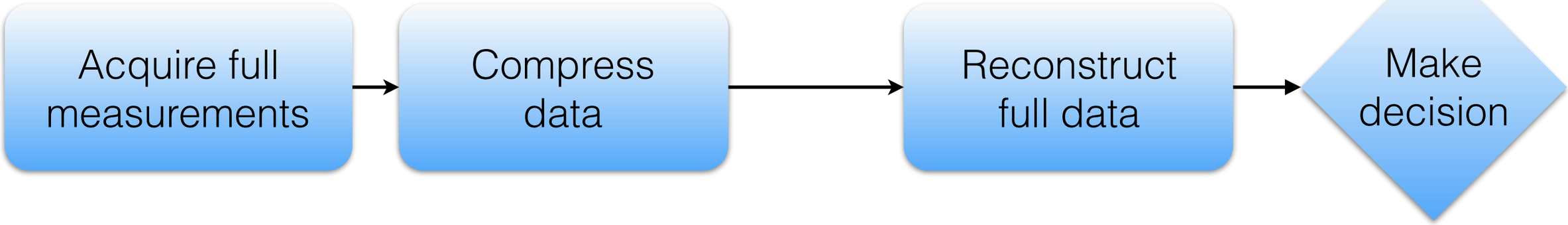




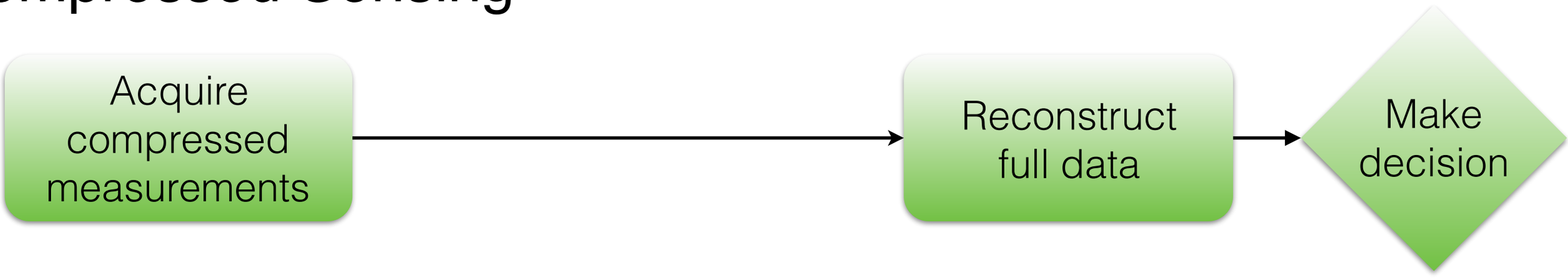


Sparse sensor placement optimization for classification (SSPOC)

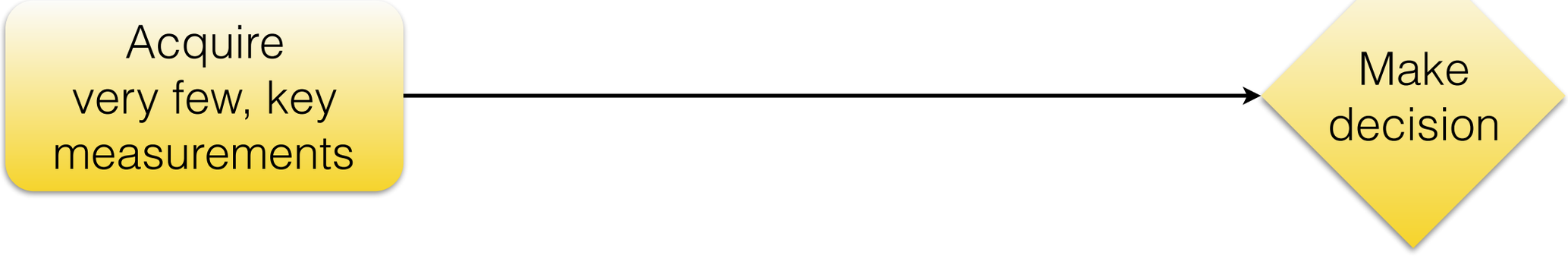
Standard

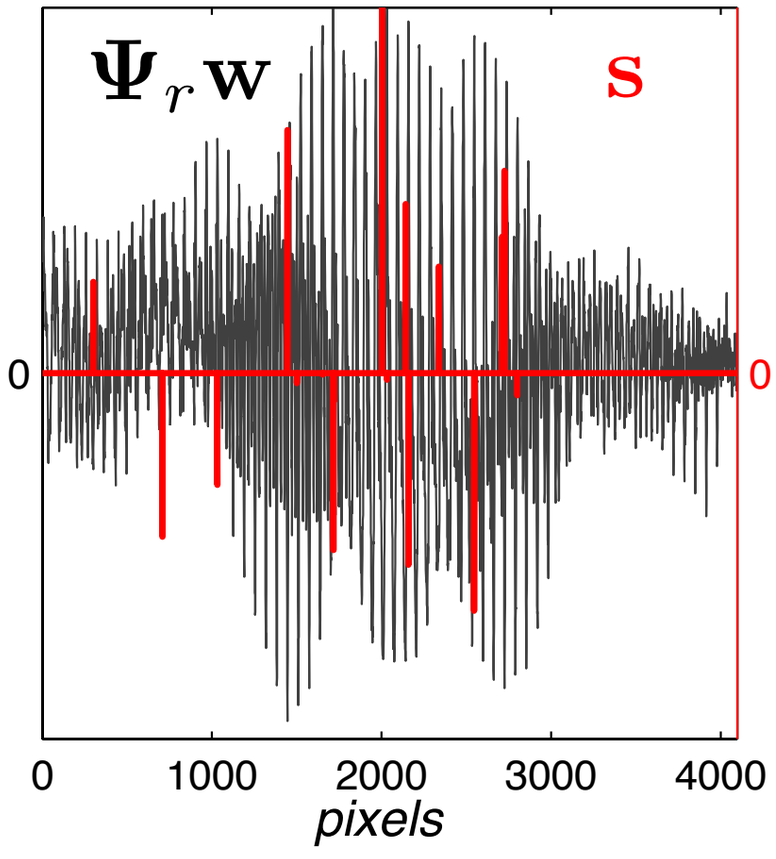


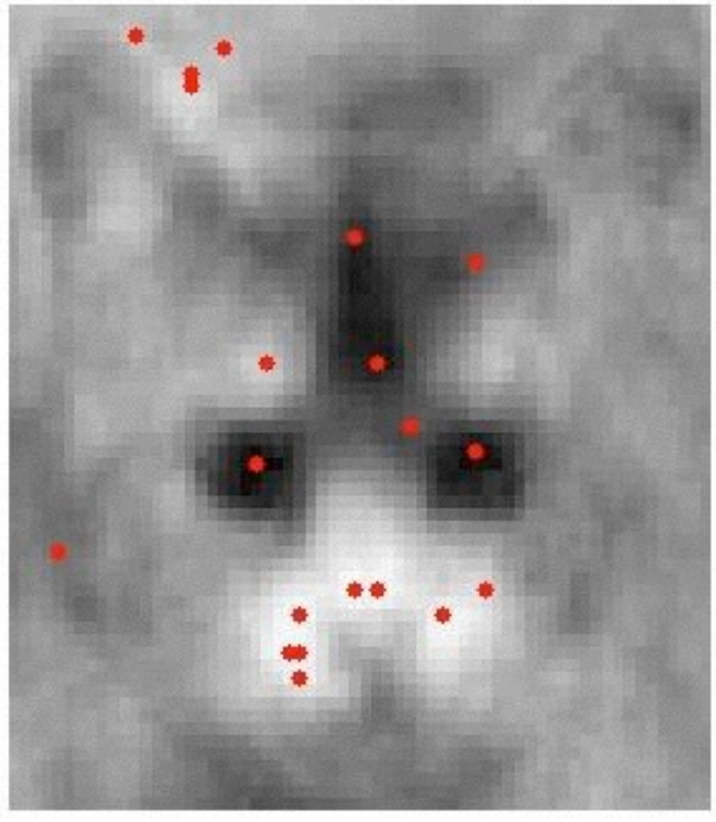
Compressed Sensing



SSPOC







$\Psi_r \mathbf{w}$

$\mathbf{s}$

from image to decision:

$$\eta = \left(\Psi_r \mathbf{w}\right)^T \mathbf{x}$$

Image has *n* pixels

$\Psi_\mathbf{r}$ feature basis, *n* × *r*

$\mathbf{w}$ decision vector, *r* × *1*

$\mathbf{s}$ sparse sensors, *n* × *1*

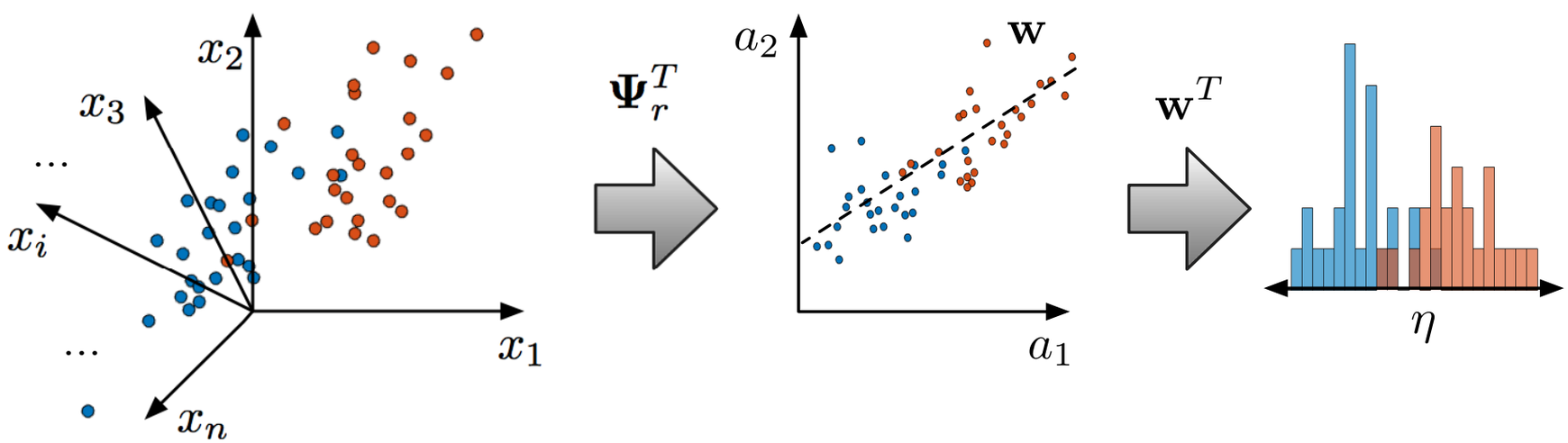
n >> *r*

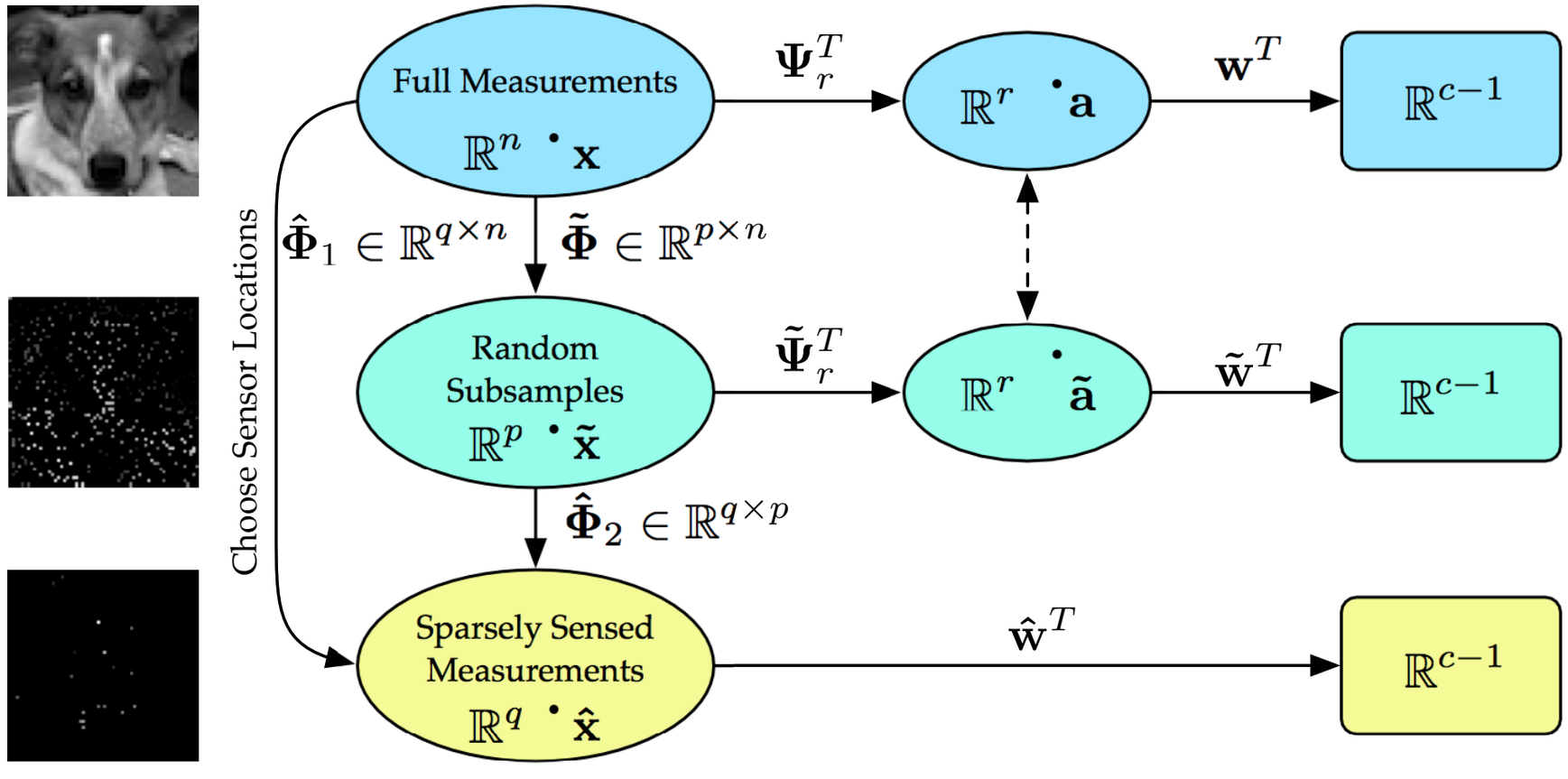
To solve for sparse sensor locations,

$$\mathbf{s} = \underset{\mathbf{s}'}{\operatorname{argmin}} ||\mathbf{s}'||_1, \quad \text{subject to } \Psi_r^T \mathbf{s}' = \mathbf{w}.$$

s is mostly zeros; the non-zero elements correspond to sensor locations, where we want to measure.

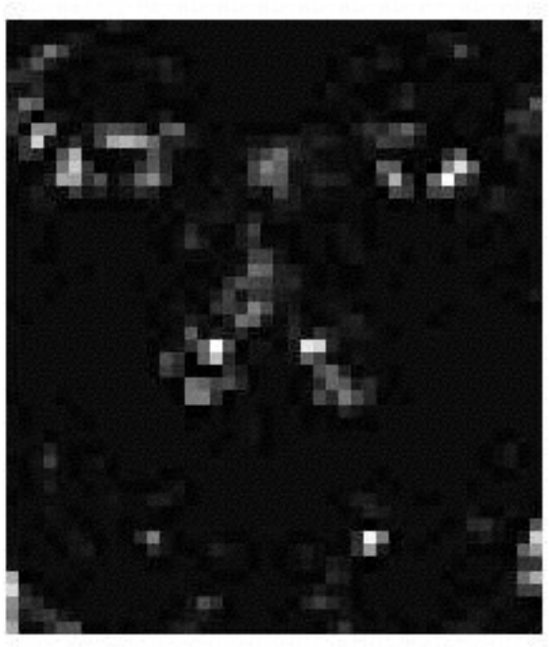
Brunton, Brunton, Proctor & Kutz, *in review*, arXiv:1310.4317.


Measurements
Features
Classification
x_2
x_3
$\cdots$
x_i
$\cdots$
x_1
x_n
Ψ_r^T
a_2
$\mathbf{w}$
a_1
$\mathbf{w}^T$
η


Full Measurements
$\mathbb{R}^n \bullet \mathbf{x}$
Ψ_r^T
$\mathbb{R}^r \bullet \mathbf{a}$
$\mathbf{w}^T$
$\mathbb{R}^{c-1}$
Choose Sensor Locations
$\hat{\mathbf{\Phi}}_1 \in \mathbb{R}^{q \times n}$
$\tilde{\mathbf{\Phi}} \in \mathbb{R}^{p \times n}$
Random Subsamples
$\mathbb{R}^p \bullet \tilde{\mathbf{x}}$
$\tilde{\Psi}_r^T$
$\mathbb{R}^r \bullet \tilde{\mathbf{a}}$
$\tilde{\mathbf{w}}^T$
$\mathbb{R}^{c-1}$
$\hat{\mathbf{\Phi}}_2 \in \mathbb{R}^{q \times p}$
Sparsely Sensed Measurements
$\mathbb{R}^q \bullet \hat{\mathbf{x}}$
$\hat{\mathbf{w}}^T$
$\mathbb{R}^{c-1}$

Which person is in the picture?

ensemble
of sparse sensor
locations

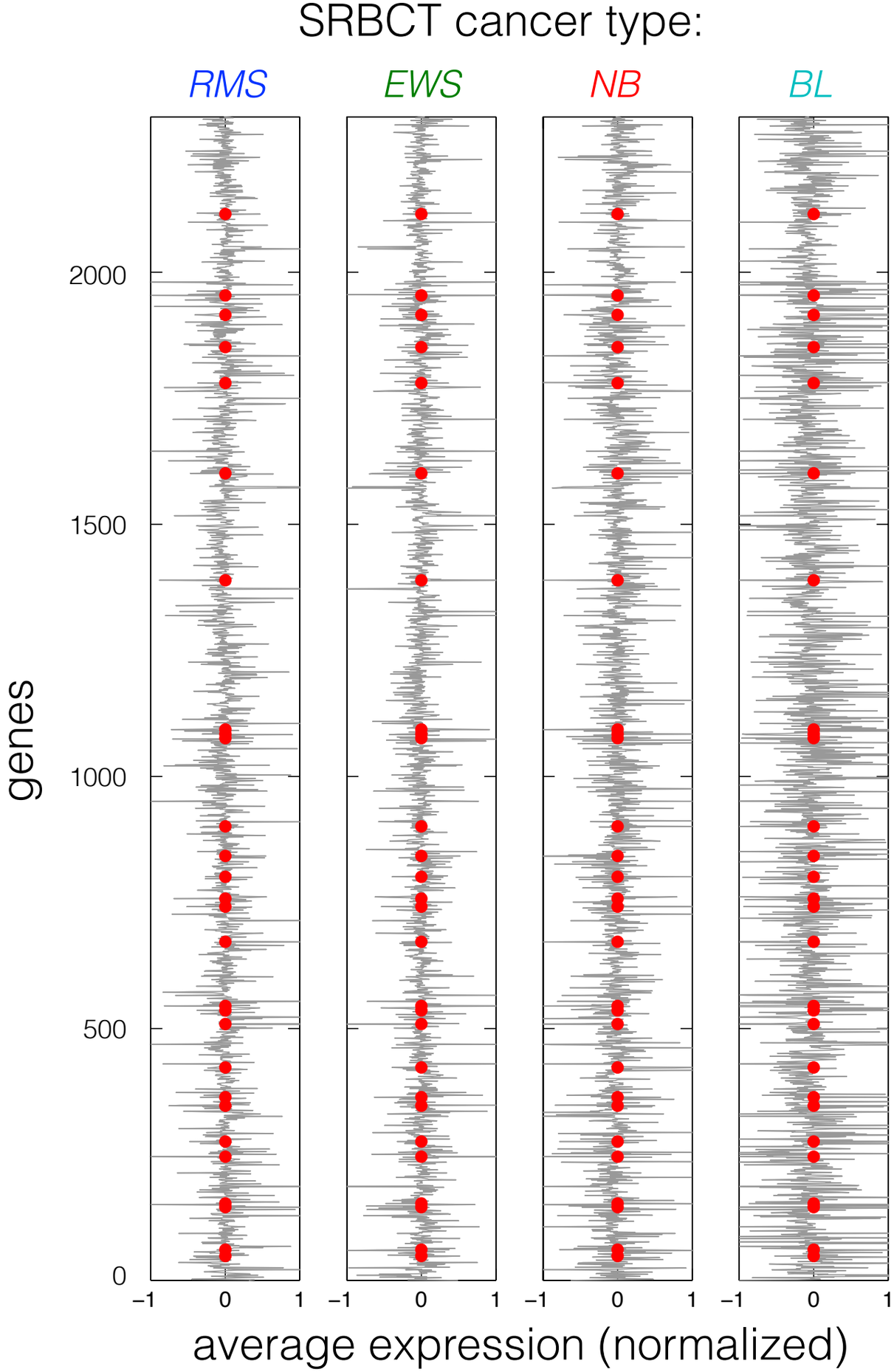


SSPOC on human faces

image

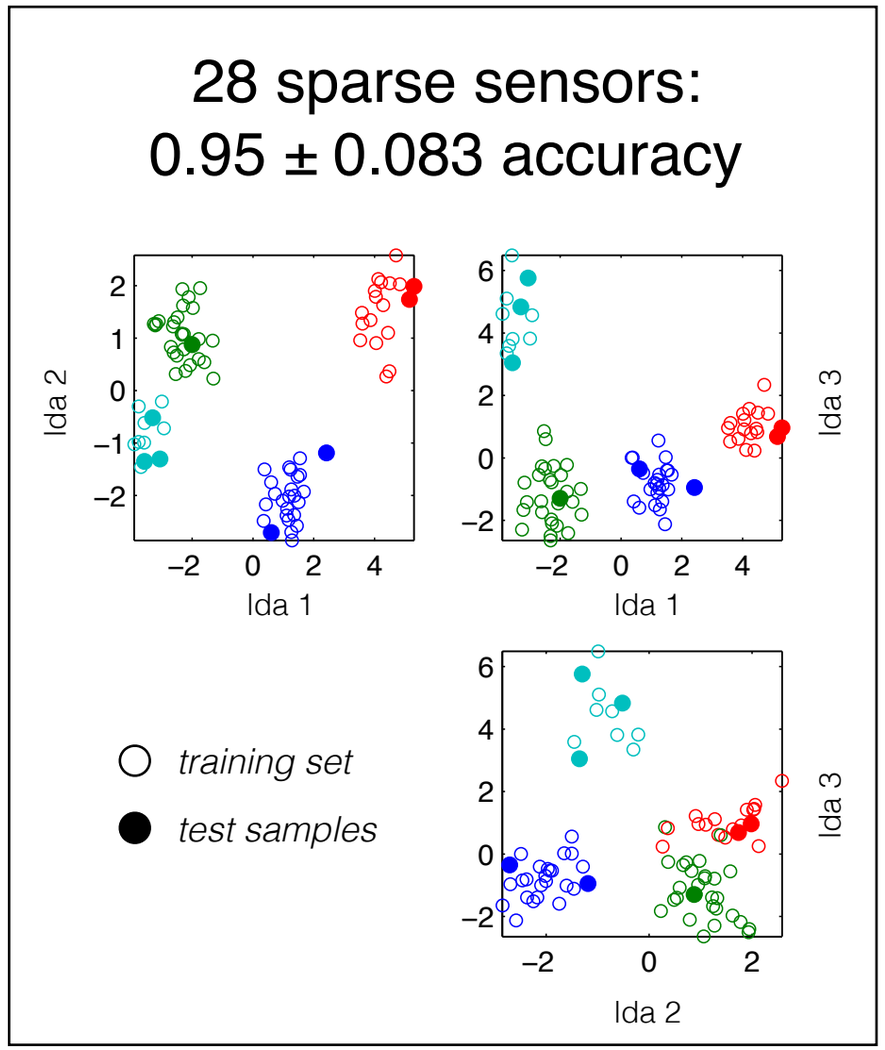
human
gaze

Yarbus, 1967.



What is the tumor type?

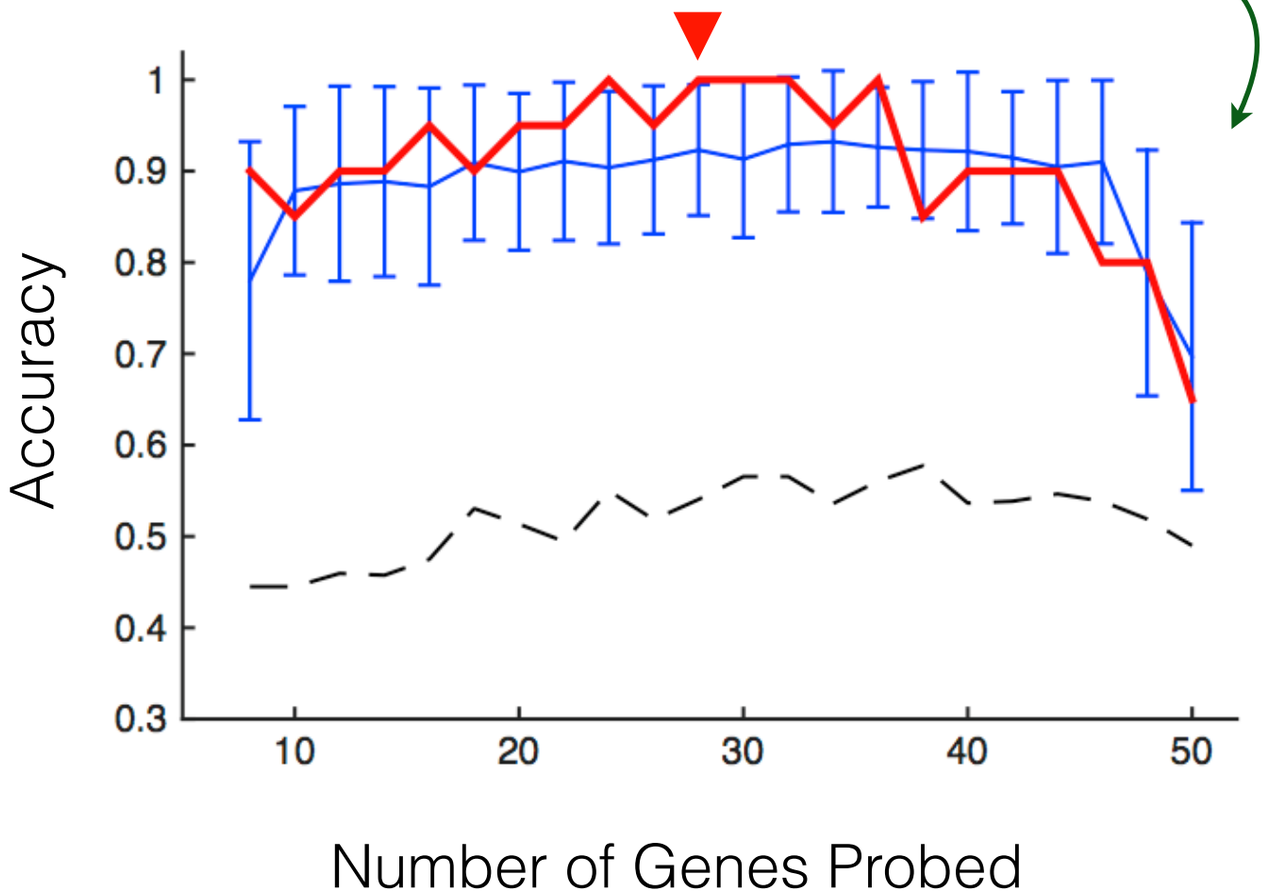
microarray dataset measured
2308 genes for 83 samples

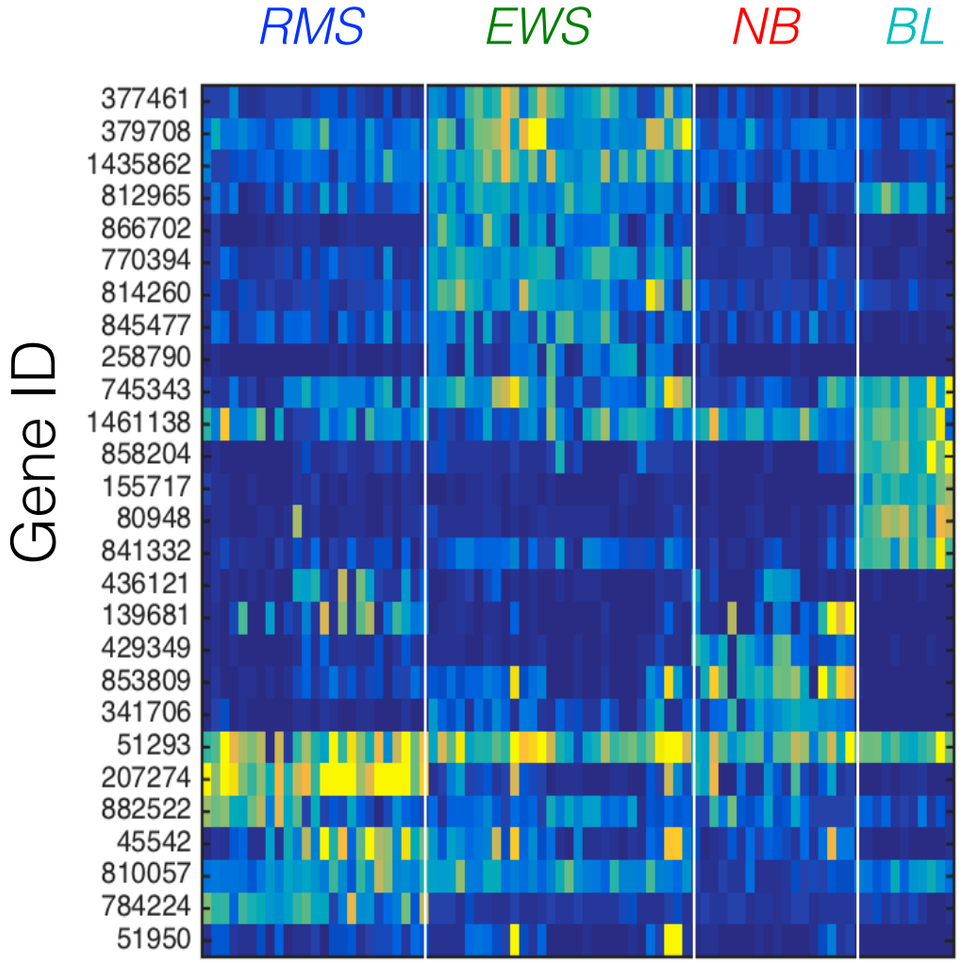


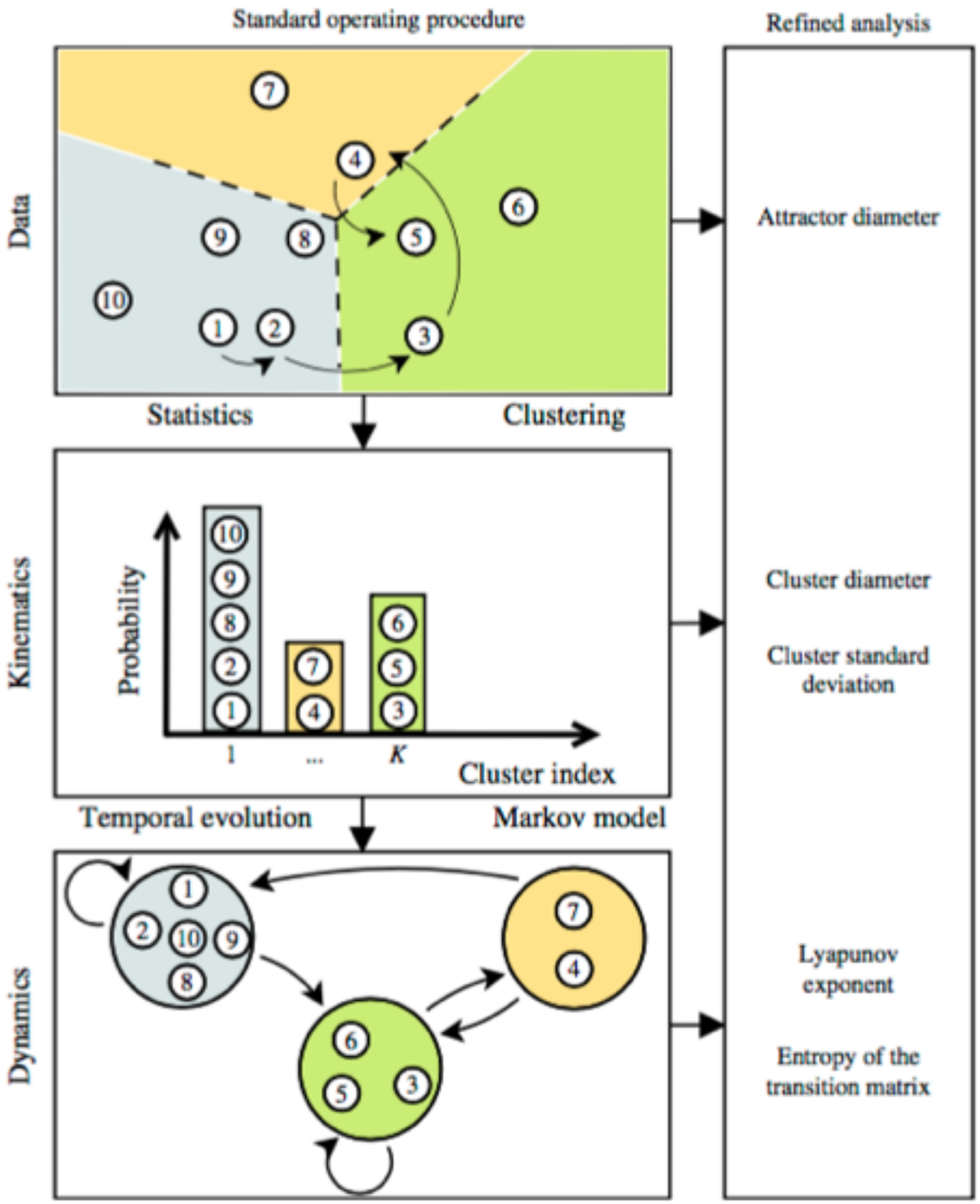
microarray data from
http://home.ccr.cancer.gov/oncology/oncogenomics/

What is the tumor type?

more measurements may
not always be better!





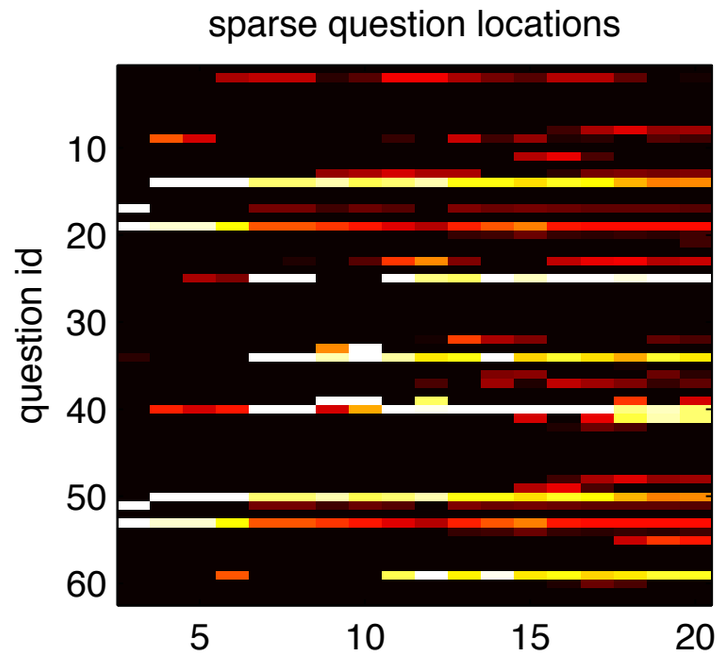


Eurika Kaiser

Probabilistic reduced order model of dynamic regimes with sparse sensors

Kaiser *et al.*, Cluster-based reduced-order modeling of a mixing layer, J Fluid Mechanics 2014.

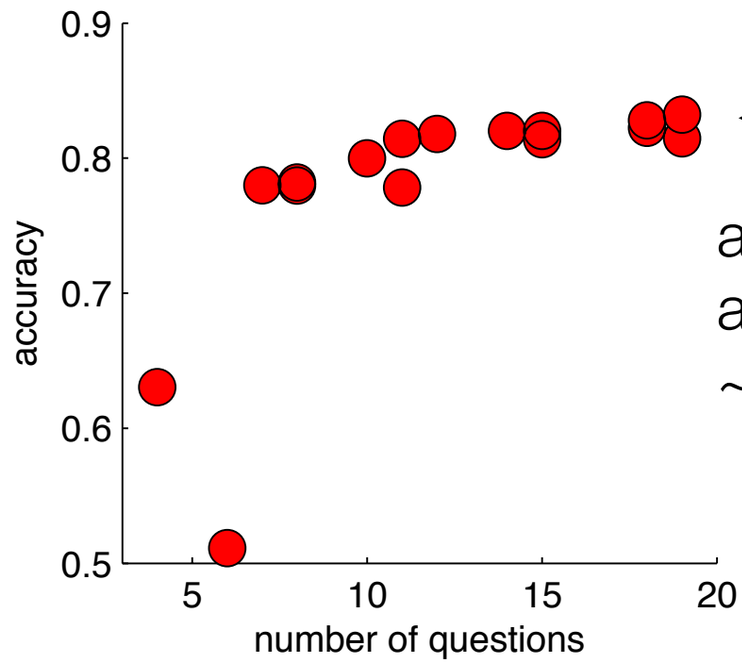
Verbal Autopsy: given a budget of questions to ask, which ones are most informative of HIV status?



questions most informative of HIV status consistently show up in solutions

Karonga VA

from ALPHA dataset



accuracy asymptotes at ~ 0.8

with C. Calvert, S. Clark, T. McCormick, UW Depts. of Sociology & Statistics

Steve Brunton
UW Mech Eng

Josh Proctor
Institute for Disease Modeling

Nathan Kutz
UW Applied Math

Washington Research FOUNDATION

ALFRED P. SLOAN FOUNDATION

AIR FORCE RESEARCH LABORATORY

NSF